Modeling the Thermal Conductivity of Concrete Based on Its Measured Density and Porosity

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ABSTRACT

The relationship between density and conductive heat transfer through multiphase, porous materials is reasonably well understood and can be evaluated using simple empirical relations. However, the complex effects of porosity, particularly pore size and volume, on heat transfer are mechanisms far less understood. To date, very few formulae are available that assess the effect that these variables have on heat conduction properties of a multiphase material.

This paper presents the results of an experimental study designed to evaluate the influence of pore characteristics on the thermal conductivity of concrete, a composite widely used as a construction material.

Twenty-one different concretes were made with densities that varied from 97 to 146 lb/ft^3 (1,550 to 2,350 kg/m^3) and porosities from 10% to 39%. The thermal conductivity of the concretes was measured using a plain hot-plate technique. Total porosity was determined using water and a vacuum saturation technique, and mercury intrusion porosimetry was used to obtain the pore size distribution.

The experimentally derived heat transfer and porosity data were used to develop a mathematical model that relates thermal conductivity to density, porosity, and median pore diameter. The model predicts values of thermal conductivity that agree closely with experimental data.

INTRODUCTION

Knowledge of the thermal conductivity of concrete is important in many areas of construction, particularly to ensure the energy-efficient design of the exterior envelopes of buildings. The thermal conducting properties of concrete that are required are normally measured under laboratory conditions using, for example, a guarded or plain hot-plate technique. Such procedures are time consuming and expensive, and they require specially trained personnel and carefully prepared samples.

Numerous formulae have been developed over the years that empirically predict the thermal conductivity of composite materials (Maxwell 1892; Eucken 1932; Brailsford and Major 1964; Reynolds and Hough 1957; Tinker 1987; Simpson and Stuckes 1986). In most cases, the thermal conductivity of a porous composite material depends on the conductivities of its component phases, the volume concentration of each, and the dispersion of the phases in the material. In order to satisfactorily model the thermal conductivity of the composites from the properties of the individual components, it is necessary to choose a model whose theoretical assumptions represent the distribution and shape of the component phases within the mixture as closely as possible. However, in view of the complexity of the structure of real materials, there are comparatively few cases where such theoretical assumptions can be applied rigorously. The limiting factor in most models is the agreement between the theoretical assumptions and the structure of the actual material.

A small number of models have been reported that specifically calculate the conductive heat transfer of concretes (Pratt 1962; Valore 1980; Campbell-Allen and Thorne 1963). These models express the conductive heat transfer in terms of the conductivities and the fractional amounts of the hydrated cement, the aggregate, and the pore phase. Only the conductive heat transfer of the entrained air is known with any certainty; the thermal conductivity of the solid component phase of the cement paste and the aggregate are unknowns. Separate measurements are particularly difficult to obtain and therefore the techniques have a severely limited application. In view of this, simplified expressions that predict the conductive heat transfer of composite materials using alternative parameters, such as density and porosity, are of particular interest.

EXPERIMENTAL PROCEDURE AND RESULTS

Specification of the Materials

Quartz, limestone, and pellite were selected as coarse aggregates to investigate the effect of density and pore characteristics on the conductive heat transfer through concretes. Quartzitic river sand was used as a fine aggregate. Quartz and limestone aggregates are natural materials, while pellite is an artificial aggregate made by pelletizing blast-furnace slag. Quartzitic coarse aggregate was used as a reference material. Limestone aggregate was chosen because it has nearly the same specific gravity and porosity as quartz but a different mineral composition. pellite aggregate was selected because it has totally different characteristics, having a lower specific gravity and a porous and glassy matrix.

Prior to any experimentation, all the coarse aggregates were air dried, sieved into single sizes, and then recom-

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bined to obtain a particle size distribution within the range recommended in British Standard 882, "Specification for Aggregates from Natural Sources for Concrete" (BSI 1986). The resultant blended grade that was used is given in Table 1.

The fine aggregate, which consisted of a quartzitic river sand, underwent a similar preparation procedure. The British Standard grading range corresponding to zone M for fine aggregates (BSI 882 1986) and the resultant size distribution are shown in Table 2. Sufficient fine pellite aggregate was also prepared to the same particle size distribution so that the pore characteristics of a pellite concrete could be studied.

Specification of the Mix Designs

Mixes were designed so that the properties of the resultant concretes would represent a wide range of porosities and densities. Variations in properties were achieved by

1. changing the cement-aggregate ratio,
2. changing the cement-sand ratio,
3. changing the water/cement ratio, and
4 adding an air-entraining agent to selected mixes.

The composition of all the mixes is given in Table 3.

Test Methods

Measurement of Conductive Heat Transfer Twelve-in.- (300-mm-) square, 2-in.- (50-mm-) thick concrete specimens were cast and then stored in an environment maintained at 100% relative humidity and 68°F (20°C) for three days before being allowed to reach their equilibrium air-dry moisture content under ambient conditions of 65% humidity and 68°F (20°C). Thermal conductivity measurements were carried out according to BSI 874 (BSI 1988) using an improved plain hot-plate apparatus designed and constructed at a British university. The apparatus consists of a central heater plate and two cold plates that sandwich the test samples. Unlike the guarded hot-plate, it has no guard ring surrounding the measurement area; however, the edges of the specimen are insulated. A correction is applied to account for any heat loss that may take place from the edges of the heater plate and specimens. The measurement uncertainty with this technique is $\pm 5\%$, and the apparatus was regularly calibrated against equipment in a British Calibration Service-accredited laboratory at a British university.

One of the main sources of error when using the unguarded hot-plate technique for measuring thermal conductivity is the contact made between the thermocouple's measurement junction and the concrete specimens. Intimate contact was achieved by rolling the thermocouples, near their hot junction, to a flatness of 0.0014 in. (0.035 mm) and applying a thin layer of heat sink compound between the measurement junction and the specimen's surface.

TABLE 1
Sieve Analysis for Coarse Aggregates

Sieve Size	BS 882 range for graded coarse aggregates Percentage by mass passing BS sieves	Blended grade used. Percentage passing
20 mm	90 - 100	100.0
14mm	54 - 75	74.8
10mm	30 - 60	45.6
5mm	0 - 10	5.0

TABLE 2
Sieve Analysis for Fine Aggregates

Sieve Size	BS 882 grading zone M for fine aggregates Percentage by mass passing BS sieves	Blended grade used. Percentage passing
1.18mm	45 - 100	87.6
600μm	25 - 100	77.8
300μm	5 - 48	26.2
150μm	0 - 10	5.0

TABLE 3
Composition of the Concrete Mixes

Mix No	Mix proportions (by mass) Cement:Sand: Agg	Type of Coarse Agg	Air-Entrained (0.26 litre per 100 kg cement)	Total Water: Cement ratio
1	1 : 2.33:3.5	Quartzitic	NO	0.53
2	"	Quartzitic	YES	0.43
3	"	Limestone	NO	0.60
4	"	Limestone	YES	0.50
5	"	Pellite	NO	0.90
6	"	Pellite	YES	0.80
7	1 : 2.33:5.6	Quartzitic	NO	0.80
8	"	Limestone	NO	0.80
9	"	Pellite	NO	0.80
10	"	Quartzitic	NO	0.56
11	"	Limestone	NO	0.66
12	"	Pellite	NO	1.05
13	1 :3.73:5.6	Quartzitic	NO	0.90
14	"	Quartzitic	YES	0.80
15	"	Limestone	NO	0.95
16	"	Limestone	YES	0.90
17	"	Pellite	NO	1.29
18	"	Pellite	YES	1.29
19	"	Quartzitic	NO	0.80
20	"	Limestone	NO	0.80
21	"	Pellite	NO	0.80

After completion of the measurement, the specimens were dried to a constant weight in a drying oven whose temperature was maintained at 221°F (105°C). Using the dry weight of the specimens, their dry density was determined. The conductive heat transfer values were adjusted to 3% moisture content by volume using a derived moisture factor equation (Tinker et al. 1989). The results are presented later in the paper.

Measurement of Total Porosity The porosity of a material is the fraction of its bulk volume occupied by voids, and, in a material such as concrete or mortar, it can be determined by measuring any two of three quantities: bulk volume, pore volume, or solid volume. In this investigation, the porosity of the concrete was obtained from bulk and pore volume quantities (Ganjian 1990) that were determined by vacuum saturation.

Essentially, vacuum saturation involves evacuating a pre-dried sample and then letting water fill the pores while the sample is still under vacuum. During the test, air in the pores of the material is removed by negative pressure and is replaced by water. After a specific period, usually between 24 and 36 hours, the pressure is increased back to atmospheric, during which time the sample reaches constant weight. For this investigation, samples 3 in. (75 mm) in diameter were cored from 4-in. (100-mm) concrete cubes, and a vacuum pressure of about 13 lb/in.^{2} (0.9 bar) was used. Porosity was calculated using the following equation, and the results that were obtained are given in Table 4:

$$P = \frac{W_s - W_d}{W_s - W_w} \times 100 , \tag{1}$$

where

P = total porosity open to water, %;
W_s = saturated sample weight in air, lb;
W_w = saturated sample weight in water, lb;
W_d = oven-dried sample weight, lb.

Measurement of Pore Size Distribution Total porosity is a parameter that does not adequately describe the pore characteristics of a concrete. A fuller specification should include pore size, shape, and structure. So that the pore size distribution of the various concretes could be investigated thoroughly, it was necessary to prepare 2-in. (50-mm) mortar cubes using the fine aggregate materials. The pore size of the voids in the mortar samples was determined by mercury intrusion porosimetry (Rootare 1970; Bakel et al. 1981). This method involves evacuating the gas from the pores and then forcing mercury into a sample by gradually increasing the pressure on the mercury. Both the volume of mercury intruded and the pressure to achieve the intrusion were measured. From these data and the knowledge of the wetting angle and surface tension of the mercury (130° and 2.8 × 10^{-4} lb/in. [48.4 dynes/cm]), respectively, the total porosity of the specimen may be determined. The cumulative pore volume against pore diameter or a pore size distribution curve may then be plotted. The instrument used in this investigation was a mercury intrusion porosimeter capable of exerting 60,000 lb/in.^{2} (414 MPa) pressure. Samples 1 in. (25 mm) in diameter were cored from the mortar cubes using a diamond saw. The cored specimens were then cut to 3/8-in. to 1/2-in.- (10- to 12-mm)-thick samples for testing. Before determining their pore size distribution, the samples were

TABLE 4
Porosity Values Obtained by Vacuum Saturation

Mix No	Mean Dry Density lb/ft.^{3} (kg/m^3)	Total Porosity (%)	Mean Total Porosity (%)
1	143 (2282)	13.08 12.78	12.9
2	131 (2091)	13.89 14.10	14.0
3	141 (2248)	14.61 15.33	15.0
4	127 (2033)	20.91 21.16	21.0
5	116 (1858)	21.44 20.63	21.0
6	97 (1557)	39.06 37.90	38.5
7	141 (2250)	14.09 13.69	13.9
8	141 (2261)	13.86 14.31	14.1
9	98 (1570)	23.42 23.33	23.4
10	146 (2341)	9.98 10.46	10.2
11	143 (2291)	11.84 12.32	12.1
12	103 (1647)	25.10 25.05	25.1
13	140 (2236)	15.38 14.91	15.1
14	126 (2020)	23.98 22.97	23.5
15	138 (2207)	17.67 17.57	17.6
16	124 (1988)	25.84 25.06	25.5
17	114 (1818)	25.57 25.46	25.5
18	97 (1551)	39.04 39.05	39.0
19	140 (2238)	13.09 12.73	12.9
20	140 (2243)	12.49 13.72	13.1
21	109 (1745)	25.86 26.24	26.1

dried at 221°F (105°C) for 24 hours to achieve constant weight. Coarse aggregates of various sizes representing each aggregate type were used to determine the pore size distribution of the aggregate phase in different concrete mixes. The mercury intrusion porosimetry results for both the mortars and coarse aggregates are given in Table 5.

From the data obtained from the mortar samples and coarse aggregates separately, it was possible to calculate the median pore diameter of the concretes. These are also shown in Table 5. A summary of the results is given in Table 6, which also includes the measured values of thermal conductivity corrected to 3% moisture content by volume.

TABLE 5
Mercury Intrusion Porosimetry Results for Mortars and Coarse Aggregates

Mortar Code Name	Relative Density (g/cc)	Total Intrusion Volume (cc/g)	Calculated Concrete Median Pore Diameter (μm)
A/0.53	2.4552	0.0807	0.0936
AE/0.43	2.4468	0.1079	0.3759
A/0.6	2.4473	0.0843	0.1367
AE/0.5	2.4218	0.1219	0.2581
PA/0.9	2.4698	0.1189	0.2790
PAE/0.8	2.4038	0.1307	0.2823
A/0.8	2.4698	0.0892	0.1348
PA/0.8	2.4018	0.0827	0.1367
B/0.8	2.4272	0.1006	0.1649
PB/0.8	2.4512	0.0789	0.1175
B/0.56	2.4580	0.0820	0.0917
B/0.66	2.4408	0.0868	0.1693
PB/1.05	2.4005	0.1093	0.3394
C/0.9	2.4881	0.1088	0.3233
CE/0.8	2.4240	0.1290	1.0057
C/0.95	2.4601	0.1213	0.3752
CE/0.9	2.5068	0.1418	1.8138
PC/1.29	2.5168	0.0995	0.2917
PCE/1.29	2.4552	0.1208	1.2760
C/0.8	2.4588	0.0944	0.3003
PC/0.8	2.4552	0.0807	0.0938

Notes:

P	=	mortar sieved from Pellite
A, B	=	Cement-sand ratio of 1:2.33
C	=	Cement-sand ratio of 1:3.73
E	=	Air-entrained mix

ANALYSIS OF RESULTS AND DEVELOPMENT OF A NEW MODEL

It has been found that a satisfactory way to quantify the pore size distribution of a concrete is to characterize the porous phase by total porosity and a calculated median pore diameter. The specification of the solid phase only included the ratios of aggregates to cement, since their influence was reflected directly in changes of porosity and median pore diameter. Increasing the percentage of quartz or limestone reduced the total porosity of the concrete because both aggregates had less pore volume than the hydrated cement and sand. Increasing the volume of pellite increased the total porosity and the median pore diameter. The effect of air entrainment was to increase the total porosity and the median pore diameter. The changes in density are the result of the combined effects of composition and pore volume.

The mechanisms by which heat is transferred through a concrete are an interrelated function of the properties of both the solid phase and the porosity. Empirical relations linking only one concrete property to thermal conductivity are adequate for a narrow range of concretes made with the same aggregate and cement phases. To develop a model that predicts thermal conductivity to an acceptable accuracy over a wide range of concretes requires that it be based on the parameters that influence the overall heat transfer process. Consequently, the interrelated data reported in this paper were used to develop a model that included the pore characteristics and the density, which, in this case, is a numerical characterization of the solid phases.

TABLE 6
Thermal Conductivity, Dry Density, Porosity, and Median Pore Diameter for the Concretes Studied

Mix No	Measured λ @ 3% mc by Vol (W/mK)	Mean Dry Density lb/ft.^{3} (kg/m^3)	Mean Total Porosity (%)	Median Pore Diameter (Micron)	Modeled Values of λ at 3% mc by Vol	% Difference from Measured Values
1	2.76	143 (2282)	12.9	0.33	2.48	-10.1
2	2.17	131 (2091)	14.0	0.48	1.94	-10.6
3	1.75	141 (2248)	15.0	0.20	1.76	0.0
4	1.50	127 (2033)	21.0	0.27	1.32	-12.0
5	0.91	116 (1858)	21.0	0.83	1.04	+14.2
6	0.65	97 (1557)	38.5	0.82	0.73	+12.3
7	2.56	141 (2250)	13.9	0.35	2.34	-8.2
8	1.73	141 (2261)	14.1	0.23	1.95	+12.7
9	0.65	98 (1570)	23.4	0.91	0.57	-12.3
10	2.86	146 (2341)	10.2	0.39	2.99	+4.5
11	1.89	143 (2291)	12.1	0.23	2.15	+13.7
12	0.69	103 (1647)	25.1	0.97	0.61	-11.5
13	2.61	140 (2236)	15.1	0.50	2.47	-5.4
14	1.99	126 (2020)	23.5	0.82	1.72	-13.6
15	1.81	138 (2207)	17.6	0.28	1.78	-1.6
16	1.44	124 (1988)	25.5	1.11	1.47	+2.0
17	0.82	114 (1818)	25.5	0.87	0.93	+13.4
18	0.63	97 (1551)	39.0	1.32	0.57	-9.5
19	2.70	140 (2238)	12.9	0.45	2.53	-6.3
20	1.96	140 (2243)	13.1	0.29	2.19	+11.7
21	0.85	109 (1745)	26.4	0.80	0.96	+12.9

The data were processed using a statistical multivariate analytical technique that resulted in the development of a polynomial expression of the following form:

$$\lambda = 1.77\times10^{-6}\rho^2 - 1139.67(\mathrm{MPD/P})^2 - 6.08\times10^{-3}\rho - 54.21(\mathrm{MPD/P}) + 6.50\times10^{-2}\rho(\mathrm{MPD/P}) + 5.47;$$

$$r = 0.962, \text{ level of significance } 99.999\%.$$

where

λ = thermal conductivity, W/m·K;
ρ = dry density, kg/m^3;
P = total porosity, %;
MPD = median pore diameter, μm.

The values of thermal conductivity predicted by the above equation are given in Table 6 together with the percentage difference from their respective measured value. It can be seen that the maximum difference is equal to $\lambda \pm 14\%$.

Further analysis revealed that the variables that can be used to predict the thermal conductivity of a concrete with a 97% confidence limit are density, porosity, and median pore diameter. Density is the most important variable in the relationship and porosity the next.

The above model contains a term involving median pore diameter, which is a difficult parameter to obtain without sophisticated equipment and trained personnel and is the least significant in the relationship. For practical purposes, a less accurate model relating thermal conductivity solely to density and total porosity is being developed.

CONCLUSIONS

1. Porosity and median pore diameter are parameters that satisfactorily represent changes in the composition and mineralogy of concrete and have been shown to be important variables that can be used to predict thermal conductivity.
2. The highly significant statistical model relating thermal conductivity to the characteristics of concrete as represented by density, porosity, and median pore diameter presented in this paper predicts values of λ within $\pm 14\%$ of the value measured to BS 874 using a plain hot-plate apparatus.
3. Using the model developed, a concrete sample could be taken from an existing structure, its dry density and porosity parameters determined, and then its thermal conductivity predicted.

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